

MAT 201 - Notes

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12.3: The Dot Product

- $$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos \theta$$

- Vectors \mathbf{u} and \mathbf{v} are orthogonal if $\mathbf{u} \cdot \mathbf{v} = 0$.

- $$\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2$$

- $$\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} \mathbf{v}$$

12.4: The Cross Product

- $$\mathbf{u} \times \mathbf{v} = (|\mathbf{u}||\mathbf{v}| \sin \theta) \mathbf{n}$$

- Nonzero vectors \mathbf{u} and \mathbf{v} are parallel iff $\mathbf{u} \times \mathbf{v} = \mathbf{0}$.

- $$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$$

- $|\mathbf{u} \times \mathbf{v}|$ is the area of a parallelogram.
- $|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|$ is the volume of a parallelepiped.

12.5: Lines and Planes in Space

- A vector equation for the line L through $P_0(x_0, y_0, z_0)$ parallel to \mathbf{v} is

$$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$$

- The plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\hat{i} + B\hat{j} + C\hat{k}$ is

$$\mathbf{n} \cdot \vec{P_0P} = 0 \text{ or } A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

– Note: $\mathbf{n} = \langle A, B, C \rangle$

- Two planes are parallel iff their normals are parallel, or $\mathbf{n}_1 = k\mathbf{n}_2$ for some scalar k .
- The angle between planes is the angle between their normal vectors.

13.1: Curves in Space and Their Tangents

- Graph 3D curves by looking only at 2 variables at a time.

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$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt}$$

- If \mathbf{r} is a differentiable vector function and the length of $\mathbf{r}(t)$ is constant, then

$$\mathbf{r} \cdot \frac{d\mathbf{r}}{dt} = 0.$$

– Note: You can prove this is true by taking the dot product of $\mathbf{r}(t) = c$ with itself.

13.2: Integrals of Vector Functions; Projectile Motion

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$$\int \mathbf{r}(t) dt = \mathbf{R}(t) + \mathbf{C}$$

13.3: Arc Length in Space

- The length of a smooth curve $\mathbf{r}(t)$, $a \leq t \leq b$ traced as t increases from a to b is

$$L = \int_a^b |\mathbf{v}| dt.$$

•

$$s(t) = \int_{t_0}^t \sqrt{[x'(\tau)]^2 + [y'(\tau)]^2 + [z'(\tau)]^2} d\tau = \int_{t_0}^t |\mathbf{v}(\tau)| d\tau$$

•

$$T = \frac{v}{|\mathbf{v}|}$$

14.1: Functions of Several Variables

- A point in a region R in the xy -plane is an **interior point** of R if a disk drawn around it is entirely in R .
- A point is a **boundary point** if a disk drawn around it lies outside of R and inside of R .
- A region is **open** if it consists entirely of interior points.
- A region is **closed** if it contains all of its boundary points.
- A region is **bounded** if it lies within a disk of finite radius.
- A region is **unbounded** if it is not bounded.

14.2: Limits

14.3: Partial Derivatives

- If partial derivatives of $f(x, y)$ exist and are continuous throughout a disk centered at (x_0, y_0) , f is continuous at (x_0, y_0) .
- If $f(x, y)$ and its partial derivatives f_x, f_y, f_{xy}, f_{yx} are defined throughout an open region containing a point (a, b) and are continuous at (a, b) , then

$$f_{xy}(a, b) = f_{yx}(a, b).$$

- If a function $f(x, y)$ is differentiable at (x_0, y_0) , then f is continuous at (x_0, y_0) .

14.4: The Chain Rule

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$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt}$$

- If instead $x = g(r, s)$, $y = h(r, s)$, and $z = k(r, s)$,

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial r} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial r}$$

– Note that it's very similar for $\frac{\partial w}{\partial s}$

- Suppose $F(x, y) = 0$ defines y as a differentiable function of x . Then at any point where $F_y \neq 0$, $\frac{dy}{dx} = -\frac{F_x}{F_y}$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

14.5: Directional Derivatives and Gradient Vectors

- The **gradient** of $f(x, y)$ is the vector

$$\nabla f = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j}$$

- If $f(x, y)$ is differentiable in an open region containing $P_0(x_0, y_0)$ then

$$D_{\mathbf{u}}f(P_0) = \nabla f(P_0) \cdot \mathbf{u}.$$

– Note that \mathbf{u} must be a unit vector.

- f increases most rapidly in the direction of the gradient vector ∇f at P . The directional derivative in this direction is $D_{\mathbf{u}}f = |\nabla f|$
- Similarly, f decreases most rapidly in the direction of $-\nabla f$, so $D_{\mathbf{u}}f = -|\nabla f|$
- Any direction \mathbf{u} orthogonal to a gradient $\nabla f \neq 0$ leads to a directional derivative of 0.
- At every point (x_0, y_0) in the domain of a differentiable function $f(x, y)$, the ∇f is normal to the level curve through (x_0, y_0) .

$$\frac{d}{dt}f(\mathbf{r}(t)) = \nabla f(\mathbf{r}(t)) \cdot \mathbf{r}'(t)$$

– Note this is very similar to the chain rule from 1D calc.

14.6: Tangent Planes and Differentials

- The **tangent plane** to the level surface $f(x, y, z) = c$ at a point $P_0 = (x_0, y_0, z_0)$ is

$$\nabla f(P_0) \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0.$$

- The **normal line** to the level surface at P_0 is

$$\langle x_0 + y_0 + z_0 \rangle + \langle f_x(P_0), f_y(P_0) + f_z(P_0) \rangle.$$

- The plane tangent to the surface $z = f(x, y)$ at $(x_0, y_0, f(x_0, y_0))$ at the point $P_0(x_0, y_0, z_0) = (x_0, y_0, f(x_0, y_0))$ is

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - z_0) = 0.$$

- Estimate the change in the value of a function f by moving a small distance ds from a point P_0 in the direction \mathbf{u} by

$$df = (\nabla f(P_0) \cdot \mathbf{u})ds$$

- The **linearization** of a function $f(x, y)$ at a point (x_0, y_0) is

$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

- Let $f(x, y)$ have continuous first and second derivatives in some rectangle R . Let M be the upper bound for $|f_{xx}|, |f_{xy}|, |f_{yy}|$ in R . Then the **error** $E(x, y)$ of the linear approximation satisfies

$$|E(x, y)| = \frac{1}{2}M(|x - x_0| + |y - y_0|)^2$$

- If we move from (x_0, y_0) to a point $(x_0 + dx, y_0 + dy)$ nearby, the total differential of f is

$$df = f_x(x_0, y_0)dx + f_y(x_0, y_0)dy.$$

14.7: Extreme Values, Saddle Points

- **First Derivative Test for Local Extrema:** If $f(x, y)$ has a local maximum or minimum at an interior point of (a, b) and if the first partial derivative exist there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

- An interior point where both f_x and f_y are zero or where both f_x and f_y and do not exist is a **critical point** of f .

– Note: Every global max/min must be a local max/min. Every local max/min must be a critical point.

- **Second Derivative Test for Local Extrema:**

– f has a **local maximum** at (a, b) if $f_{xx} < 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b) .

– f has a **local minimum** at (a, b) if $f_{xx} > 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b) .

– f has a **saddle point** at (a, b) if $f_{xx}f_{yy} - f_{xy}^2 < 0$ at (a, b) .

– The test is **inconclusive** if $f_{xx}f_{yy} - f_{xy}^2 = 0$.

14.8: Lagrange Multipliers

- Suppose that $f(x, y, z)$ and $g(x, y, z)$ are differentiable and $\nabla g \neq \mathbf{0}$ when $g(x, y, z) = 0$. To find the local maximum and minimum values of f subject to the constraint $g(x, y, z) = 0$, find the values of x, y, z and λ that satisfy

$$\nabla f = \lambda \nabla g \text{ and } g(x, y, z) = 0.$$

- Remember to check the boundaries.

14.9: Taylor's Formula for Two Variables

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$$f(x, y) = f(0, 0) + xf_x + yf_y + \frac{1}{2}(x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy})$$

•

$$|E| \leq \frac{M}{3!}(|x - x_0| + |y - y_0|)^3$$

15.1: Double and Iterated Integrals over Rectangles

- **Fubini's Theorem:** If $f(x, y)$ is continuous throughout a region R , then $\iint f(x, y) dx dy = \iint f(x, y) dy dx$.

– Note that if R is rectangular, we can switch the bounds without redrawing the region R .

15.2: Double Integrals over General Regions

- Fubini's Theorem also applies just make sure you change your bounds of integration.
- $dy dx$: Sketch the region. Draw an arrow parallel to the y axis in the direction of $+y$. Where it enters is its lower bound. Where it leaves is its upper bound. The x -limits are the ones that include all vertical arrows you can draw through region R .
- $dx dy$: Same thing as above but draw arrows parallel to x axis instead.

15.3: Area by Double Integration

- The **area** of a closed, bounded region R is

$$A = \iint_R dA.$$

- The **average value** of f over R is

$$\frac{1}{\text{Area}} \iint_R f \, dA.$$

15.4: Double Integrals in Polar Form

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$$\iint_R f(r, \theta) \, dA = \iint f(r, \theta) \, r \, dr \, d\theta.$$

- Change of coordinates works the same as for § 15.3.

15.5: Triple Integrals in Rectangular Coordinates

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$$V = \iiint_D dV$$

- The **average value** of F over D is

$$\frac{1}{\text{volume of } D} \iiint_D f \, dV$$

15.6: Applications of Triple Integrals

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$$M = \iiint_D \text{density}(x, y, z) \, dV$$

- Center of mass:

$$\bar{x} = \iiint_D x \, \text{density}(x, y, z) \, dV,$$

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$$\bar{y} = \iiint_D y \, \text{density}(x, y, z) \, dV$$

•

$$\bar{z} = \iiint_D z \, \text{density}(x, y, z) \, dV$$

15.7: Triple Integrals in Cylindrical and Spherical Coordinates

- **Cylindrical Coordinates:**

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z, \quad \tan \theta = y/x$$

– Remember:

$$dV = r \, dz \, dr \, d\theta$$

and θ always comes from the x-axis

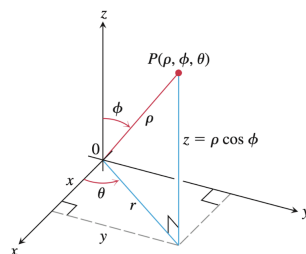
• **Spherical Coordinates:**

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi$$

– Remember:

$$dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

and $0 \leq \phi \leq \pi$



15.8: Substitutions in Multiple Integrals

- The **Jacobian** of the coordinate transformation $x = g(u, v)$ and $y = h(u, v)$ is

$$J(u, v) = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

- Under the transformation $x = g(u, v)$ and $y = h(u, v)$,

$$\iint_R f(x, y) \, dx \, dy = \iint_G f(g(u, v), h(u, v)) \frac{\partial(x, y)}{\partial(u, v)} \, du \, dv$$

- For triple integrals, it's basically the same thing.

16.1: Line Integrals of Scalar Functions

- If f is defined on a curve C given parametrically by $\mathbf{r}(t) = \langle g(t), h(t), k(t) \rangle$, then the line integral is

$$\int_a^b f(g(t), h(t), k(t)) |\mathbf{v}(t)| \, dt$$

16.2: Vector Fields and Line Integrals: Work, Circulation, and Flux

- The **gradient field** of a differentiable function $f(x, y, z)$ is the field of gradient vectors

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

- Evaluate the line integral of $F = M(x, y, z)\hat{i} + N(x, y, z)\hat{j} + P(x, y, z)\hat{k}$ along $C: \mathbf{r}(t) = \langle g(t), h(t), k(t) \rangle$:

- Substitute $x = g(t)$, $y = h(t)$, $z = k(t)$ into $M(x, y, z)$, $N(x, y, z)$ and $P(x, y, z)$ of \mathbf{F} .
- Find the derivative velocity vector $\frac{d\mathbf{r}}{dt}$.
- Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \frac{d\mathbf{r}}{dt} dt$$

- Line integrals with respect to dx, dy, dz :

–

$$\int_C M dx + N dy + P dz,$$

where $\int_C M(x, y, z) dx = \int_a^b M(g(t), h(t), k(t)) g'(t) dt$, and so on for N and P .

- The **flow** along the curve is just the line integral along that curve.
- The **flux** of a vector field $\mathbf{F} = M(x, y)\hat{i} + N(x, y)\hat{j}$ if \mathbf{n} is the outward pointing normal vector is

$$\int_C \mathbf{F} \cdot \mathbf{n} = \oint_C M dy - N dx$$

- If the motion is counterclockwise, $\mathbf{n} = \mathbf{T} \times \mathbf{k}$. If it's clockwise, $\mathbf{n} = \mathbf{k} \times \mathbf{T}$, where \mathbf{k} is the unit vector in the z -direction and $\mathbf{T} = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$.

16.3: Path Independence, Conservative Fields, and Potential Functions

- If the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ along all paths C from A to B is the same, then the integral is **path independent** and the field \mathbf{F} is **conservative**.
- If \mathbf{F} is a vector field defined on open region D and $\mathbf{F} = \nabla f$ for some scalar function f on D , then f is a **potential function for \mathbf{F}** .

- Line integrals in conservative fields:

$$\int_A^B \mathbf{F} \cdot d\mathbf{r} = \int_A^B \nabla f \cdot d\mathbf{r} = f(B) - f(A)$$

- If $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ around every closed curve in D , the field \mathbf{F} is conservative and vice versa.
- Let $\mathbf{F} = M(x, y, z)\mathbf{i} + N(x, y, z)\mathbf{j} + P(x, y, z)\mathbf{k}$ be over an open, simply connected domain. Then, \mathbf{F} is conservative iff

$$P_y = N_z, \quad M_z = P_x, \quad \text{and} \quad N_x = M_y.$$

- Maybe the thing about exact differential forms....?

16.4: Green's Theorem in the Plane

- The **circulation density** ($\text{curl } \mathbf{F} \cdot \mathbf{k}$) of a vector field $\mathbf{F} = M\mathbf{i} + N\mathbf{j}$ at the point (x, y) is

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}.$$

- The **flux density (divergence)** of a vector field $\mathbf{F} = M\mathbf{i} + N\mathbf{j}$ at the point (x, y) is

$$\text{div } \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}.$$

- **Green's Theorem:**

$$\oint_C \mathbf{F} \cdot \mathbf{T} \, ds = \oint_C M \, dx + N \, dy = \iint_R (\text{curl } \mathbf{F} \cdot \mathbf{k}) \, dx \, dy$$

$$\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \oint_C M \, dy - N \, dx = \iint_R (\text{div } \mathbf{F}) \, dx \, dy$$

16.5: Surfaces and Areas

- A surface is parametrized by

$$\mathbf{r}(u, v) = f(u, v)\mathbf{i} + g(u, v)\mathbf{j} + h(u, v)\mathbf{k}$$

- The **area** of a smooth surface parametrized by \mathbf{r} , $a \leq u \leq b$, $c \leq v \leq d$ is

$$A = \iint_R |\mathbf{r}_u \times \mathbf{r}_v| \, dA = \int_c^d \int_a^b |\mathbf{r}_u \times \mathbf{r}_v| \, du \, dv$$

- The **surface area differential** is

$$d\sigma = |\mathbf{r}_u \times \mathbf{r}_v| du dv$$

- In spherical coordinates, it's useful to remember $d\sigma = |\mathbf{r}_\phi \times \mathbf{r}_\theta| = R^2 \sin \phi d\theta d\phi$

- The area of the surface $F(x, y, z) = c$ over a closed and bounded plane region R is

$$\iint_R \frac{|\nabla F|}{|\nabla F \cdot \mathbf{p}|} dA,$$

where $\mathbf{p} = \mathbf{i}, \mathbf{j},$ or \mathbf{k} is normal to R and $\nabla F \cdot \mathbf{p} \neq 0$.

- For a graph $z = f(x, y)$ over a region R in the xy plane, the surface area formula is

$$A = \iint_R \sqrt{f_x^2 + f_y^2 + 1} dx dy$$

16.6: Surface Integrals

Formulas for a Surface Integral of a Scalar Function

1. For a smooth surface S defined **parametrically** as $\mathbf{r}(u, v) = f(u, v)\mathbf{i} + g(u, v)\mathbf{j} + h(u, v)\mathbf{k}$, $(u, v) \in R$, and a continuous function $G(x, y, z)$ defined on S , the surface integral of G over S is given by the double integral over R ,

$$\iint_S G(x, y, z) d\sigma = \iint_R G(f(u, v), g(u, v), h(u, v)) |\mathbf{r}_u \times \mathbf{r}_v| du dv. \quad (2)$$

2. For a surface S given **implicitly** by $F(x, y, z) = c$, where F is a continuously differentiable function, with S lying above its closed and bounded shadow region R in the coordinate plane beneath it, the surface integral of the continuous function G over S is given by the double integral over R ,

$$\iint_S G(x, y, z) d\sigma = \iint_R G(x, y, z) \frac{|\nabla F|}{|\nabla F \cdot \mathbf{p}|} dA, \quad (3)$$

where \mathbf{p} is a unit vector normal to R and $\nabla F \cdot \mathbf{p} \neq 0$.

3. For a surface S given **explicitly** as the graph of $z = f(x, y)$, where f is a continuously differentiable function over a region R in the xy -plane, the surface integral of the continuous function G over S is given by the double integral over R ,

$$\iint_S G(x, y, z) d\sigma = \iint_R G(x, y, f(x, y)) \sqrt{f_x^2 + f_y^2 + 1} dx dy. \quad (4)$$

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- The **flux** (or **surface integral**) of a vector field \mathbf{F} over a smooth surface S having chosen normal unit vectors \mathbf{n} is

$$\iint_S \mathbf{F} \cdot \mathbf{n} d\sigma.$$

– The outward normal is given by

$$\hat{\mathbf{n}} = \frac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|}.$$

So,

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iint_R \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, du \, dv$$

– If S is a part of a level surface $g(x, y, z) = c$, then

$$\mathbf{n} = \pm \frac{\nabla g}{|\nabla g|},$$

so

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iint_R \mathbf{F} \cdot \frac{\pm \nabla g}{|\nabla g \cdot \mathbf{p}|} \, dA.$$

16.7: Stokes' Theorem

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$$\text{curl} \mathbf{F} = \nabla \times \mathbf{F}$$

- **Stokes' Theorem:** Let $\mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$. Then, the circulation of \mathbf{F} around boundary curve C in the counterclockwise direction with respect to the surface's unit normal vector \mathbf{n} equals the integral of the curl vector field over S , a piecewise smooth surface:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, d\sigma$$

– Right hand rule! Curl fingers in the direction of C counterclockwise and your thumb is the normal vector.

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$$\text{curl grad} = \mathbf{0} \quad \text{or} \quad \nabla \times \nabla f = \mathbf{0}$$

16.8: The Divergence Theorem

- The **divergence** of a vector field $\mathbf{F} = M(x, y, z)\mathbf{i} + N(x, y, z)\mathbf{j} + P(x, y, z)\mathbf{k}$ is

$$\text{div} \mathbf{F} = \nabla \cdot \mathbf{F}.$$

- The flux of vector field \mathbf{F} across a piecewise smooth oriented closed surface S in the direction of the surface's outward unit normal \mathbf{n} equals the triple integral of the divergence of \mathbf{F} over the region D enclosed by the surface:

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iiint_D \nabla \cdot \mathbf{F} \, dV$$

- For every vector field \mathbf{F} ,

$$\operatorname{div}(\operatorname{curl}\mathbf{F}) = 0$$

Green's Theorem and Its Generalization to Three Dimensions

Tangential form of Green's Theorem: $\oint_C \mathbf{F} \cdot \mathbf{T} \, ds = \iint_R (\nabla \times \mathbf{F}) \cdot \mathbf{k} \, dA$

Stokes' Theorem: $\oint_C \mathbf{F} \cdot \mathbf{T} \, ds = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, d\sigma$

Normal form of Green's Theorem: $\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_R \nabla \cdot \mathbf{F} \, dA$

Divergence Theorem: $\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iiint_D \nabla \cdot \mathbf{F} \, dV$

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